Branch : Mechanical engg.
Semester :- 4 th

Code - 1625405
Sub :- Fluid mechanics and Machinery

Unit - 07

— : Pumps : —

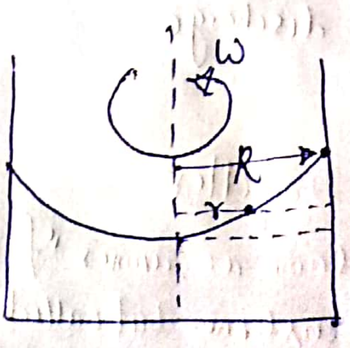
Aim :- To transfer the mechanical energy of motor to the fluid in the form of pressure energy and Kinetic energy.

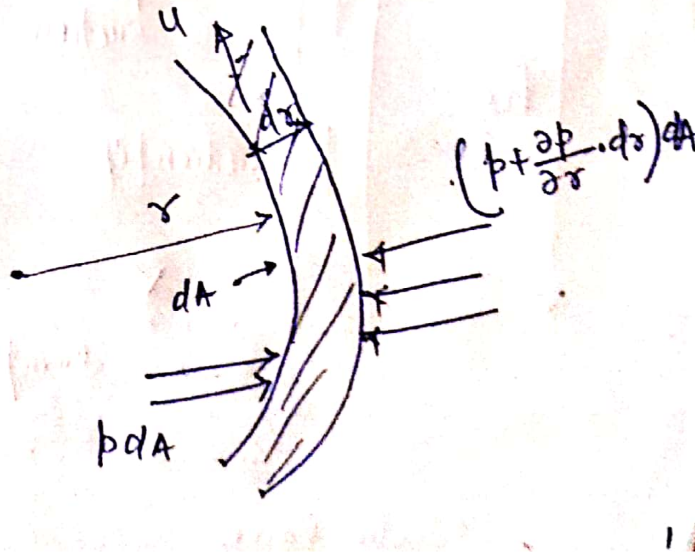
Centrifugal pump :— If the mechanical energy is converted into pressure energy by means of centrifugal force acting on the fluid, the hydraulic machine is called centrifugal pump.

Principle of centrifugal pump :-

The centrifugal pump acts as a reverse of an inward radial flow reaction turbine. This means that the flow in centrifugal pumps is in the radial outward directions. The centrifugal pump works on the principle of forced vortex flow.

When the mass of the fluid is rotating about an axis, then there is rise in pressure in radial outward direction. The rise in pressure is directly proportional to square of tangential velocity at that point.





By force balance,

$$\cancel{\left(p+\frac{\partial p}{\partial r}\cdot dr\right)} \cancel{dA} - \cancel{p\,dA} = \left(\rho\, \cancel{dA}\, \cancel{dr}\right)\cdot\frac{u^2}{r}$$

$m v^2 / r$

$$\frac{\partial p}{\partial r} = \frac{\rho u^2}{r}$$

$$\frac{\partial p}{\partial r} = \rho\frac{\omega^2 r^2}{r}\cdot\partial r \qquad [\because u = \omega r]$$

$$\partial p = \rho\omega^2 r\, dr$$

$$p_2 - p_1 = \frac{\rho\omega^2(r_2^2 - r_1^2)}{2}$$

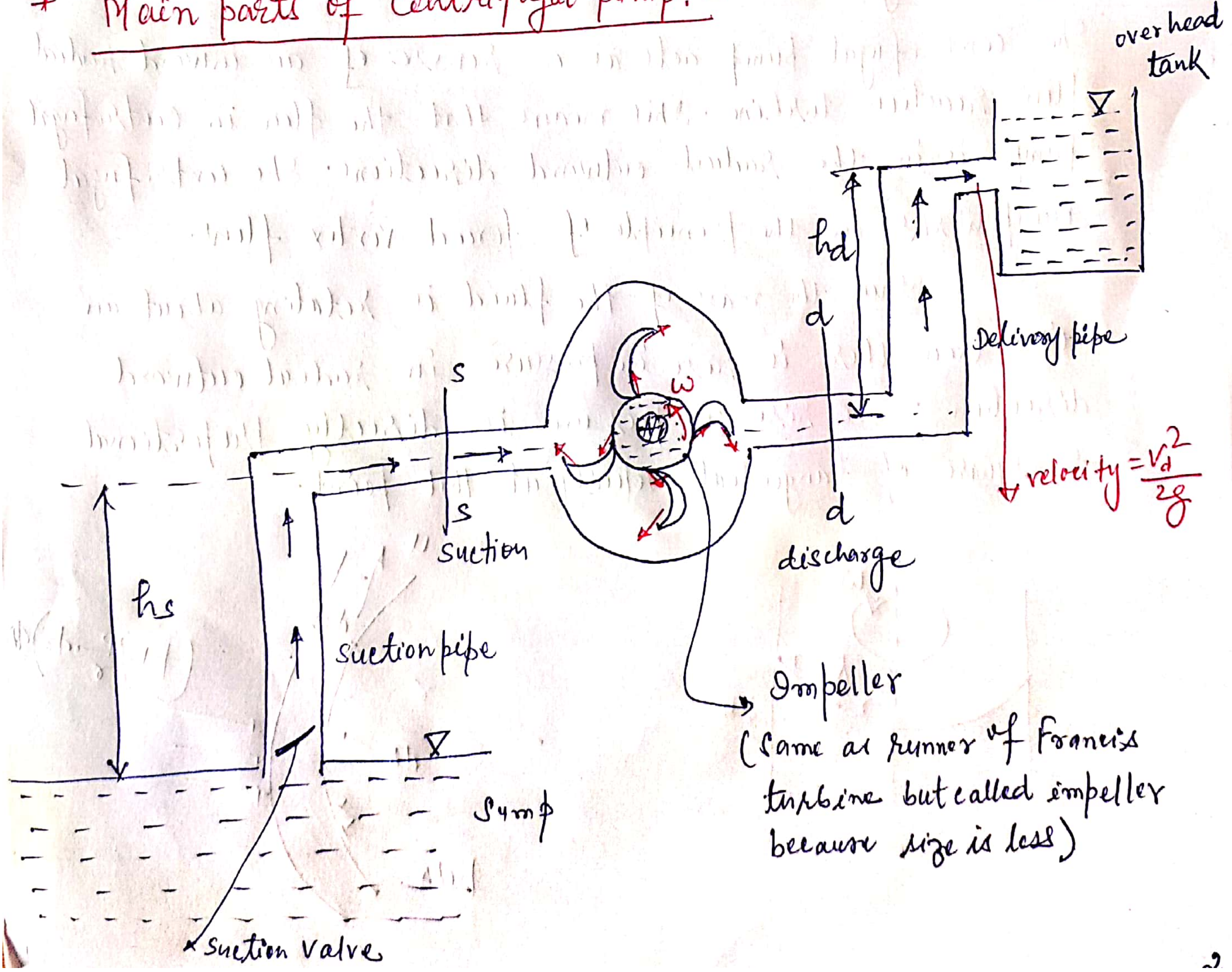
$$\boxed{p_2 - p_1 = \frac{\rho(u_2^2 - u_1^2)}{2}}$$

$$\boxed{\frac{p_2 - p_1}{\rho g} = \frac{u_2^2 - u_1^2}{2g} = H_{generated}}$$

$u_2 > u_1 \quad [\because d_2 > d_1]$

change in pressure head due to centrifugal action.

* Main parts of centrifugal pump:-



(same as runner of Francis turbine but called impeller because size is less)

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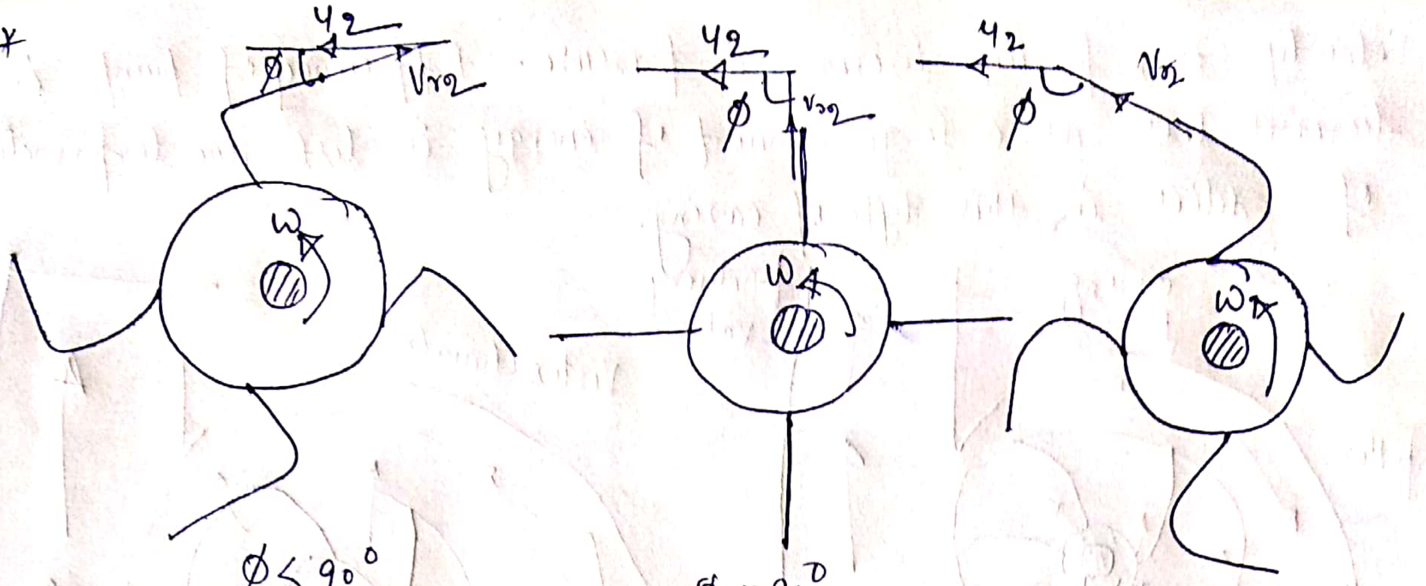


$\phi < 90^\circ$ (Backward Vane)

$\phi = 90^\circ$ (radial Vane)

$\phi > 90^\circ$ (forward Vane)

* Main parts of centrifugal pump: —

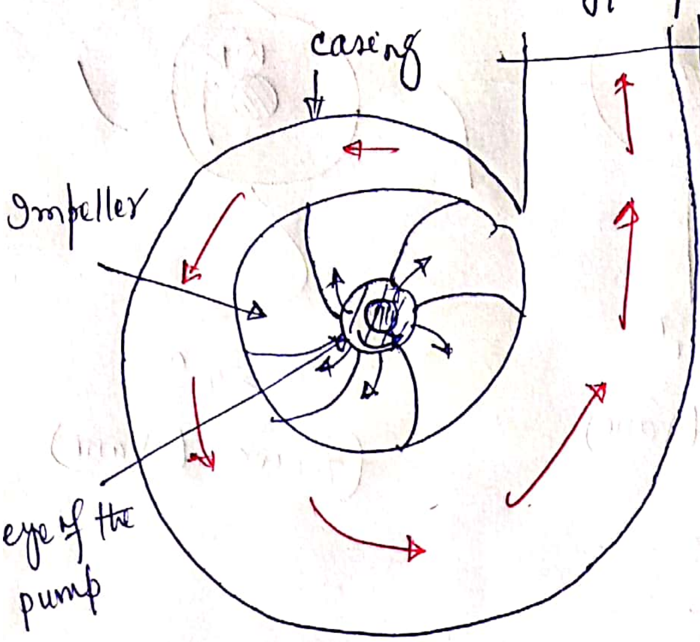
1. Impeller
2. Casing
3. Suction pipe with a foot valve and strainer
4. Delivery pipe

1. **Impeller :-** The rotating part of a centrifugal pump is called impeller. It consists of a series of backward curved vanes. The impeller is mounted on a shaft which is further connected to the shaft of an electric motor.

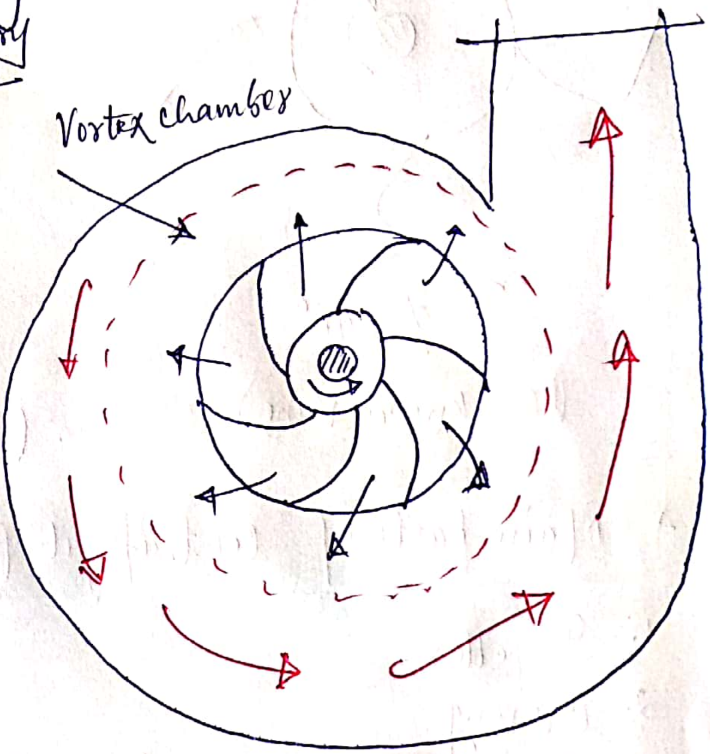
2. **Casing :-** The casing of centrifugal pump is similar to casing of a reaction turbine. It is an air tight passage surrounding the impeller and is designed in such a way that the kinetic energy of the water discharged at the outlet of the impeller is converted into pressure energy before the water leaves the casing and enters the delivery pipe.

Following three types of casings are commonly adopted:

(a) **Volute casing** — It is of spiral type in which area of flow increases gradually. Increase in area causes decrease in velocity. thus finally increase in pressure. Efficiency of pump is ~~decreases~~ less as a large amount of energy is lost due to formation of eddies in this type of casing.



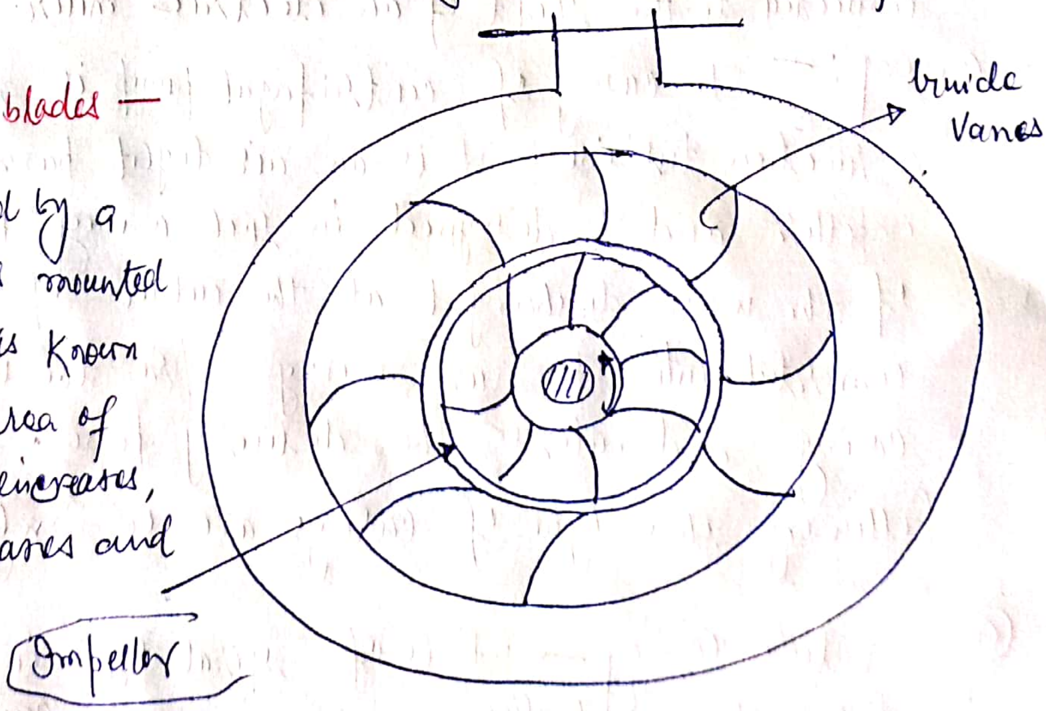
(a) volute casing



(b) Vortex casing

(b) Vortex casing — If a circular chamber is introduced between the casing and the impeller as shown in figure, the casing is known as Vortex casing. loss of energy due to formation of eddies is reduced by considerable amount. It's efficiency is more than that of volute casing.

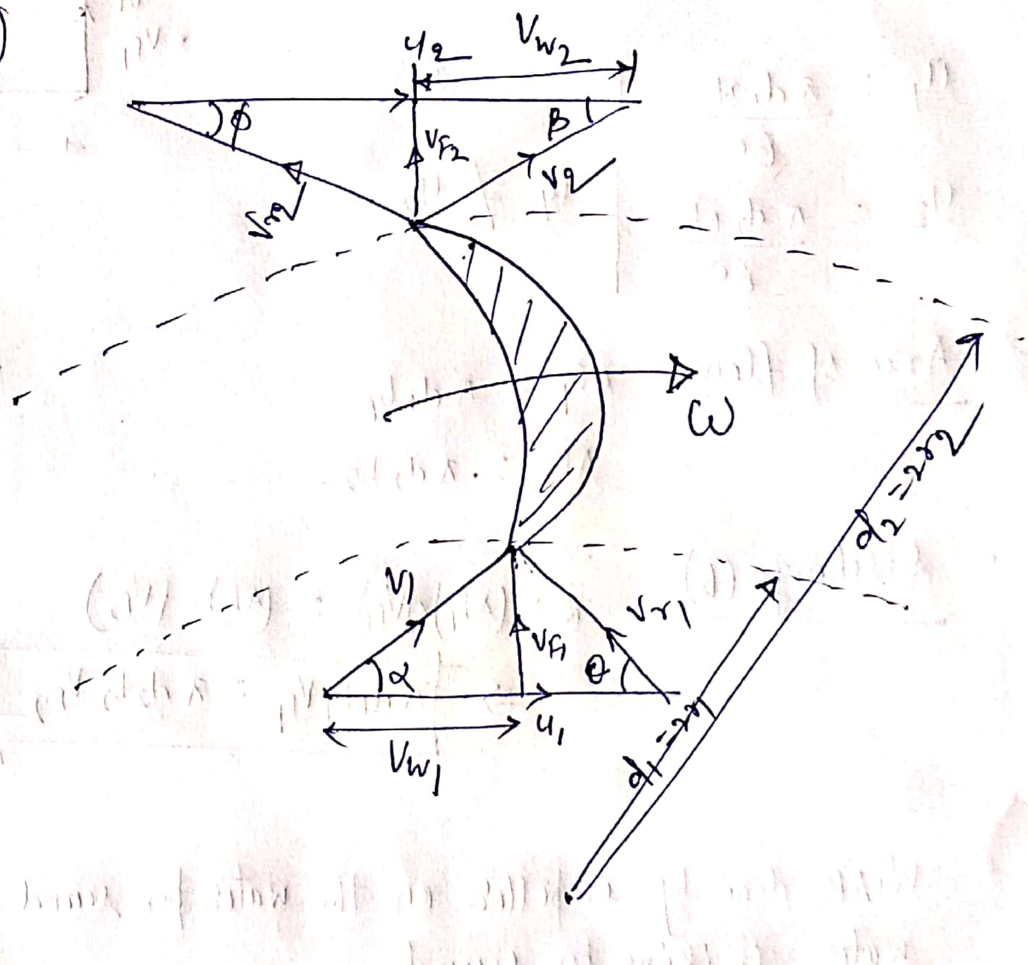
(c) Casing with guide blades — Impeller is surrounded by a series of guide blades mounted on a ring which is known as diffuser. The area of the guide vanes ~~as~~ increases, thus velocity decreases and pressure increases.



3. Suction pipe — A pipe whose one end is connected to the inlet of the pump and other ends dips into water in a sump is known as suction pipe. A foot valve which is a one way type of valve is fitted at the lower end of the suction pipe. A strainer is also fitted at the lower end of the suction pipe.

4. Delivery pipe — A pipe whose one end is connected to the outlet of the pump and other ends delivers the water at a required height is known as delivery pipe.

Analysis (In general)

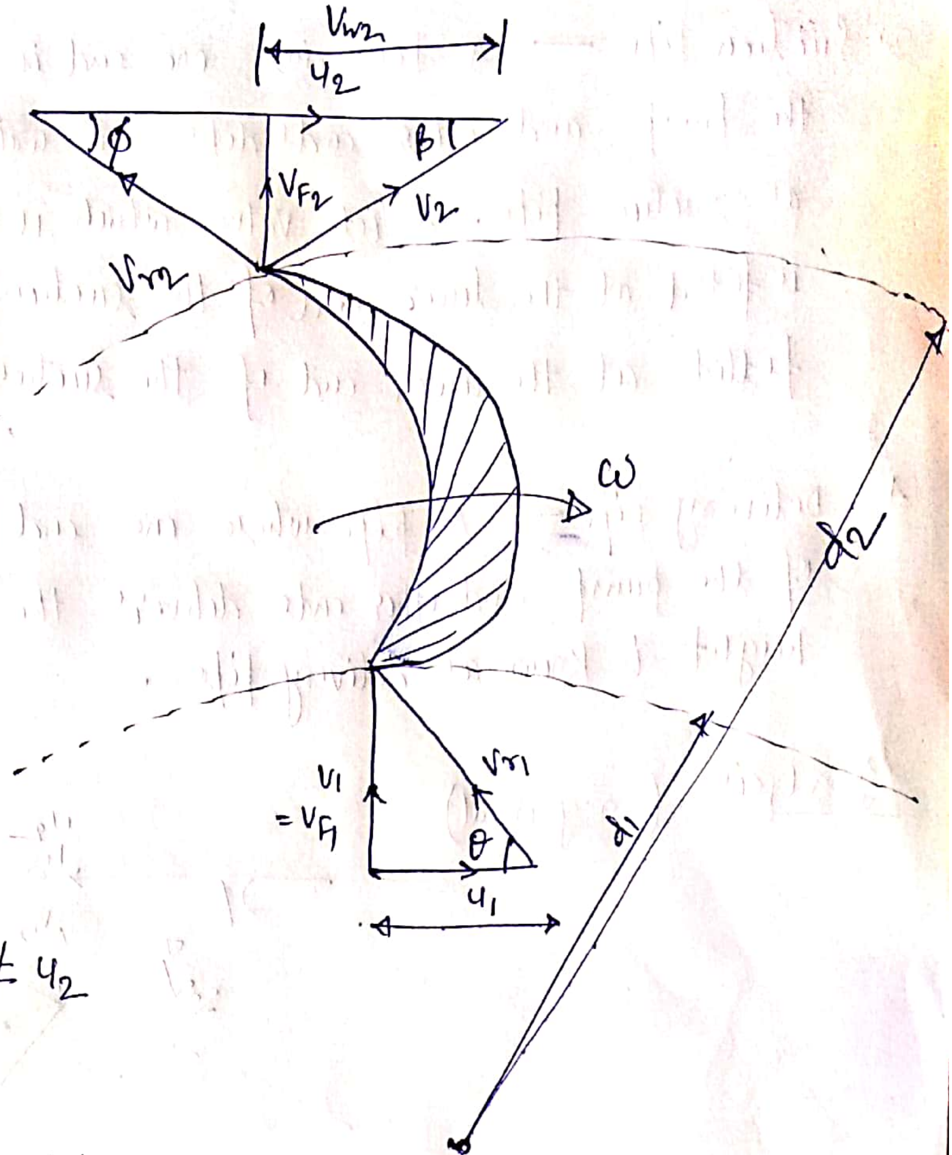


Centrifugal pump

$\alpha = 90^\circ$

$V_{w_1} = 0$

$V_1 = V_{F_1}$



$$u_1 = \frac{\pi d_1 N}{60}$$

$$u_2 = \frac{\pi d_2 N}{60}$$

$u_1 \neq u_2$

- **Area of flow**: $A_{F_1} = \pi d_1 b_1$

  $A_{F_2} = \pi d_2 b_2$

- **Discharge (Q)**: $Q = (A_{F_1}) \times (V_{F_1}) = (A_{F_2}) \times (V_{F_2})$

  $$\Rightarrow \boxed{\pi d_1 b_1 V_{F_1} = \pi d_2 b_2 V_{F_2}}$$

- Work done by impeller on the water per second per unit weight of water striking per second

  $= -$ [Work done in case of inward flow reaction turbine]

  $$= -\left[\frac{1}{g}\left(V_{w_1} u_1 - V_{w_2} u_2\right)\right] = \frac{1}{g}\left[V_{w_2} u_2 - V_{w_1} u_1\right]$$

But here, $V_{w_1} = 0 \Rightarrow$ work done by impeller on the water per second $= \dfrac{V_{w_2} u_2}{g}$

• **Manometric head:** It is the net energy given by pump to water or it is the head against which pump is working. It can be calculated in following ways—

h_s = Suction head to pump

h_d = delivery head

H_s = static head

$$\boxed{H_s = h_s + h_d}$$

h_{fs} = Head loss in suction pipe

h_{fd} = Head loss in delivery pipe

$$\boxed{H_f = h_{fs} + h_{fd}}$$

V_d = discharge velocity

Then

$$\boxed{\text{manometric head } (H_m) = \underbrace{H_s}_{(h_s + h_d)} + \underbrace{H_f}_{(h_{fs} + h_{fd})} + \frac{V_d^2}{2g}}$$

If V_d is too small or not given in question then neglected.

$$\therefore \boxed{H_m = H_s + H_f}$$

again,

$$H_m = \left(\begin{matrix}\text{total energy of fluid} \\ \text{at exit of pump}\end{matrix}\right) - \left(\begin{matrix}\text{total energy of fluid} \\ \text{at inlet of pump}\end{matrix}\right)$$

$$H_m = \left(\frac{P_d}{\rho g} + \frac{V_d^2}{2g} + Z_d\right) - \left(\frac{P_s}{\rho g} + \frac{V_s^2}{2g} + Z_s\right)$$

Here, $Z_d \simeq Z_s$

$$\therefore H_m = \left(\frac{P_d}{\rho g} + \frac{V_d^2}{2g}\right) - \left(\frac{P_s}{\rho g} + \frac{V_s^2}{2g}\right)$$

Now, d_s = diameter of suction pipe

d_d = diameter of delivery pipe

If $d_s = d_d \Rightarrow$ then $V_s = V_d$ then, $\boxed{H_m = \frac{P_d}{\rho g} - \frac{P_s}{\rho g}}$

* If friction / mechanical loss is taken into account, then –

$$\boxed{H_m = \left(\frac{V_{w_2} u_2}{g}\right) - (\text{Impeller loss}) + (\text{recovery in casing})}$$

* Power: –

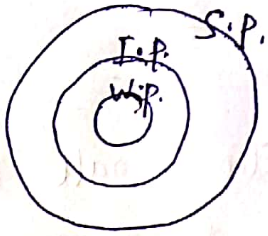
- Water power or Mechanical power = $\boxed{\rho g Q H_m}$.
- Impeller power (IP) = –(Rotor power of inward reaction turbine)

$$IP = -\left[\rho Q(V_{w_1}u_1 - V_{w_2}u_2)\right], \quad \underline{\text{and } V_{w_1} = 0.}$$

$$\boxed{IP = \rho Q(V_{w_2}u_2).}$$

- $\boxed{\text{Shaft power (SP)} = \text{I.P.} + \text{Mechanical losses}}$

$$\boxed{S.P. > I.P. > W.P.}$$



Efficiency: –

- $\boxed{\eta_{volumetric} = \frac{Q}{Q + \Delta Q}}$, Q = volume per second, discharge from delivery pipe

  ΔQ = leakage loss

  (generally leakage occurs in delivery pipe, because pressure is more)

- $\eta_{manometric} = \frac{W.P.}{I.P.} = \frac{\cancel{\rho} g \cancel{Q} H_m}{\cancel{\rho Q}(V_{w_2}u_2)}$

$$\boxed{\eta_{mano} = \frac{g H_m}{V_{w_2}u_2}}$$

- $\boxed{\eta_{mechanical} = \frac{I.P.}{S.P.}}$

- $\eta_{overall} = \frac{W.P.}{S.P} = \frac{WP}{IP} \times \frac{IP}{SP}$

$$\boxed{\eta_o = \eta_{mano.} \times \eta_{mech.}}$$

- If $\eta_{volumetric}$ is given –

  then $\boxed{\eta_o = \eta_{mano} \times \eta_{mech} \times \eta_{vol.}}$

- Speed ratio $(K_u) = \dfrac{u_2}{\sqrt{2gH_m}}$
- Flow ratio $(K_f) = \dfrac{V_{f_2}}{\sqrt{2gH_m}}$
- Diameter ratio $= \dfrac{d_1}{d_2} = 0.5$

Note —
Here,
speed ratio and flow ratio both are taken at exit of pump. But in case of turbine it is taken at inlet of turbine.

* Minimum speed required for working of pump: —

- The pump will deliver the water when,

$$H_{generated} \geq H_{required} \, (H_m).$$

$$\Rightarrow \frac{u_2^2 - u_1^2}{2g} \geq H_m$$

$$\Rightarrow \omega^2 (r_2^2 - r_1^2) \geq 2gH_m$$

$$\Rightarrow \boxed{\omega = \sqrt{\frac{2gH_m}{r_2^2 - r_1^2}} = \frac{2\pi N}{60}}$$

This equation gives the minimum starting speed of the centrifugal pump.

* Concept of multistage in centrifugal pump.

(a) Multistage centrifugal pumps for high heads: —

For developing a high head, a number of impellers are mounted in series or on the same shaft.

- The water from suction pipe enters the 1st impeller at inlet and is discharged at outlet with increased pressure. The water with increased pressure from the outlet of the 1st impeller is taken to the inlet of the 2nd impeller with the help of a connecting pipe.
- At the outlet of the 2nd impeller, the pressure of water will be more than the pressure of water at outlet of 1st impeller.

$\therefore$ total head developed $= \boxed{n \times H_m \text{ of each pump.}}$

where, n = Number of identical impellers mounted on the same shaft
H_m = Head developed by each impeller

- When impellers are mounted in series, then discharge remains same. $\boxed{Q = \text{constant.}}$

(b) Multistage centrifugal pumps for high discharge :—

For obtaining high discharge, the pump should be connected in parallel. Each of the pumps lifts the water from a common pump and discharges water to a common pipe to which the delivery pipes of each pump is connected. Each of the pump is working against the same head.

$$\boxed{H_m = \text{constant}}$$

Total discharge $(Q)_{Total} = \boxed{n \times \text{discharge of each pump}}$

where, n = Number of identical pumps arranged in parallel.

Priming of Pump

- If there is a leak in suction valve, then casing and the pipe will get empty and the pump stops working because air ~~exits~~ enters the casing. When we restart the pump, the impeller will rotate in air and generate the head in terms of meter of air column. We know that density of air is very-very less compared to water, therefore, generated head may not be sufficient to lift the water. Therefore, before restarting the pump, the casing has to be filled by water. This process is called priming of pump.

* **Cavitation :-** Cavitation is defined as the formation of vapour bubbles of the flowing liquid and collapsing of the vapour bubbles. Formation of vapour bubbles of the flowing liquid takes place only whenever the pressure in any region falls below vapour pressure. When the pressure of the flowing liquid is less than its vapour, the liquid starts boiling and vapour bubbles are formed. These vapour bubbles are carried along with the flowing liquid to higher pressure zones, then bubbles collapse. Due to sudden collapsing of the bubbles on the metallic surface, high pressure is produced causing damage of the metallic surfaces.

- **Area of occurance of cavitation —**

1. In reaction turbine — In reaction turbine the cavitation may occur at the outlet of the runner or at the inlet of the draft tube where the pressure is considerably reduced.

2. In centrifugal pump — In centrifugal pumps the cavitation may occur at the inlet of the impeller of the pump or at the suction side of the pumps, where the pressure is considerably reduced.

- **Effects of cavitation —**

1. Due to sudden collapse of vapour bubbles, considerable noise and vibrations are produced.
2. The metallic surfaces are damaged and cavities are formed on the surfaces.
3. The efficiency of a turbine decreases due to cavitation.

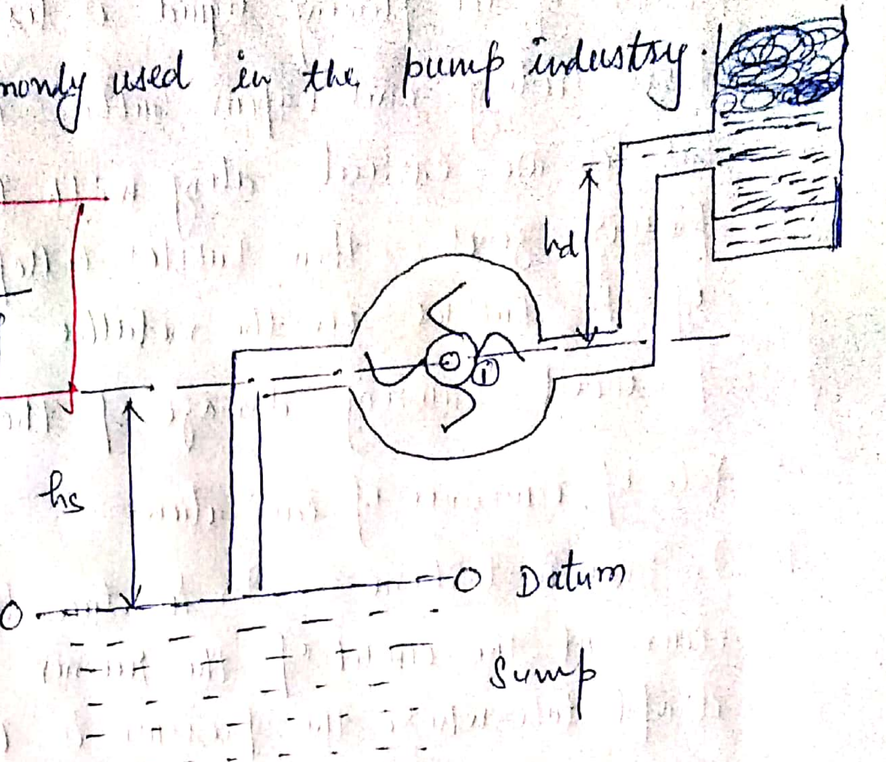
- **Methods to avoid cavitation —**

1. To maintain the fluid pressure more than vapour pressure of fluid at that temperature.
2. By providing the coating of materials such as Aluminium-bronze and stainless steel is coated over the vane surface.

Net positive suction head (NPSH)

- It is the difference between pump inlet stagnation pressure head (sum of absolute pressure head and velocity head) and vapour pressure head.
- The term NPSH is very commonly used in the pump industry.

$$\boxed{NPSH = \left(\frac{p_1}{\rho g} + \frac{v_1^2}{2g}\right) - \frac{p_v}{\rho g}}$$



Applying Bernoulli's equation between 0-0 and 1-1.

$$\frac{p_{atm}}{\rho g} + 0 + 0 = \frac{p_1}{\rho g} + \frac{v_1^2}{2g} + h_s$$

$$\left(\frac{p_1}{\rho g} + \frac{v_1^2}{2g}\right) = \left(\frac{p_{atm}}{\rho g} - h_s\right) \quad \text{--- (1)}$$

$$\therefore NPSH = \frac{p_{atm}}{\rho g} - h_s - \frac{p_v}{\rho g}$$

$$\boxed{NPSH = h_a - h_s - h_v}$$

atmospheric pressure head — suction head — vapour pressure head

It is fixed for a manufactured motor

Now. $\frac{p_{atm}}{\rho g} + 0 + 0 = \frac{p_1}{\rho g} + \frac{v_1^2}{2g} + h_s$

Here, if h_s is fixed, then after increasing v_1, pressr is decreasing. But pressure should not less than vapour pressure, otherwise cavitation starts. So, $\frac{p_1}{\rho g}$ should be kept maximum.

* Thomas cavitation factor (σ) to avoid cavitation —

$$\boxed{\sigma = \frac{NPSH}{H_m}}$$

To avoid cavitation, $\boxed{\sigma > \sigma_c}$ → Critical value

* Specific speed of centrifugal pump — (N_s) :

It is the speed of geometrically similar pump which would deliver unit discharge (one cubic meter) per second when working against unit head (one meter). It is denoted by N_s.

$Q = 1\ m^3/s$

$H = 1\ m$

$Q = aV$

$Q \propto D^2 V$ → can't be taken as constant

$u \propto \sqrt{H_m}$

$u \propto DN$

$D \propto \frac{u}{N} \propto \frac{\sqrt{H_m}}{N}$

$V \propto \sqrt{H_m}$

$\therefore\ Q \propto \left(\frac{\sqrt{H_m}}{N}\right)^2 \cdot \sqrt{H_m}$

$Q \propto \frac{H_m^{3/2}}{N^2}$

$$\Rightarrow \boxed{\frac{N^2 Q}{H_m^{3/2}} = K = \text{constant}}$$

By definition,

$Q = 1\ m^3/s$

$H = 1\ m$

then $N = N_s$

$\frac{N_s^2 \times 1}{1^{3/2}} = K = \text{constant}$

$N_s^2 = K = \frac{N^2 Q}{H_m^{3/2}}$

$$\boxed{N_s = \frac{N\sqrt{Q}}{H_m^{3/4}}}$$

* Model - prototype relations for centrifugal pump —

① $\boxed{\frac{gH_m}{N^2D^2} = \text{constant}}$ $\Rightarrow$ $\left(\frac{\sqrt{H_m}}{ND}\right)_{model} = \left(\frac{\sqrt{H_m}}{ND}\right)_{prototype}$

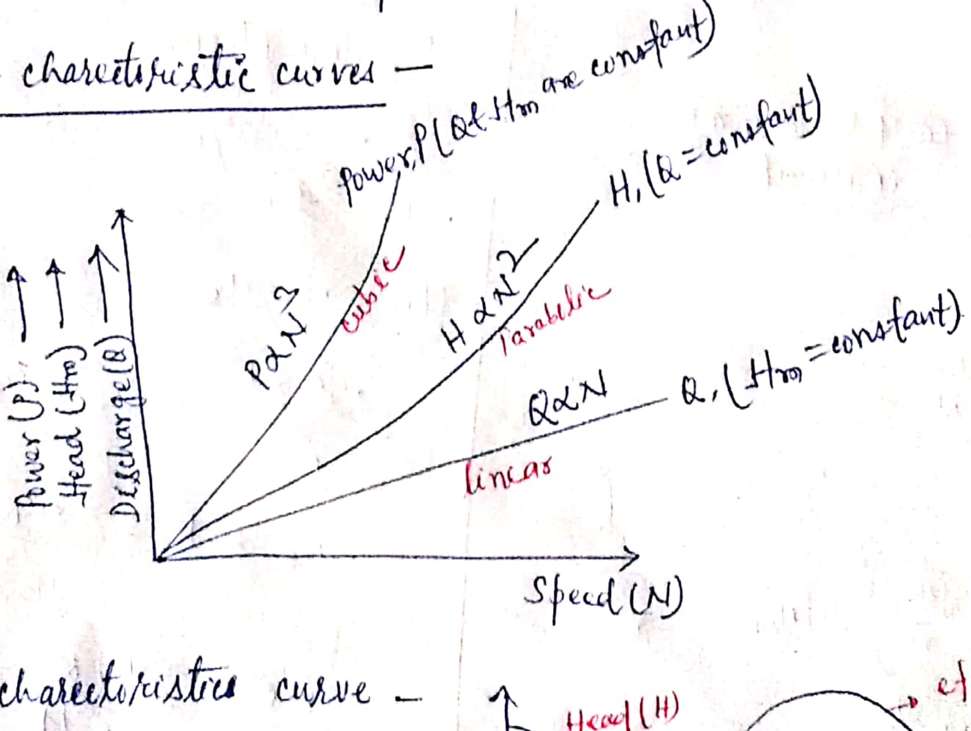
② $\boxed{\frac{Q}{ND^3} = \text{constant}}$ $\Rightarrow$ $\left(\frac{Q}{ND^3}\right)_m = \left(\frac{Q}{ND^3}\right)_p$

③ $\boxed{\frac{P}{N^3D^5} = \text{constant}}$ $\Rightarrow$ $\left(\frac{P}{N^3D^5}\right)_m = \left(\frac{P}{N^3D^5}\right)_p$

* Characteristic curves of centrifugal pumps —

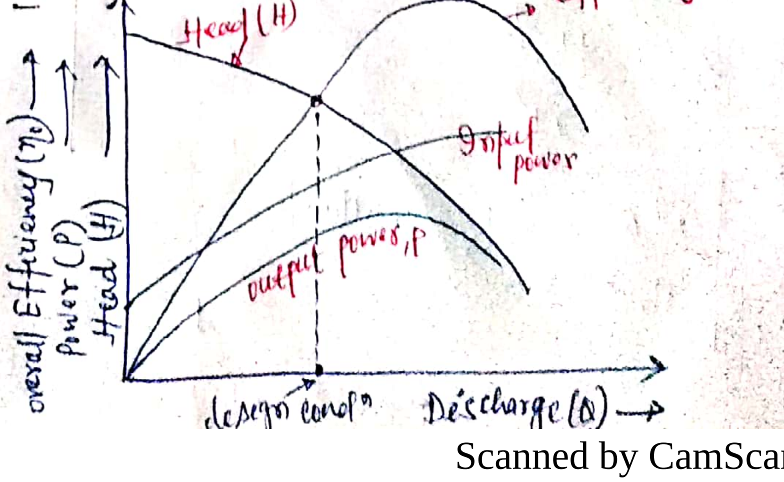
- These curves which are plotted from the results of a number of tests on the centrifugal pump. These curves are necessary to predict the behaviour and performance of the pump when pump is working under different flow rate, head and speed.

1. Main characteristic curves —

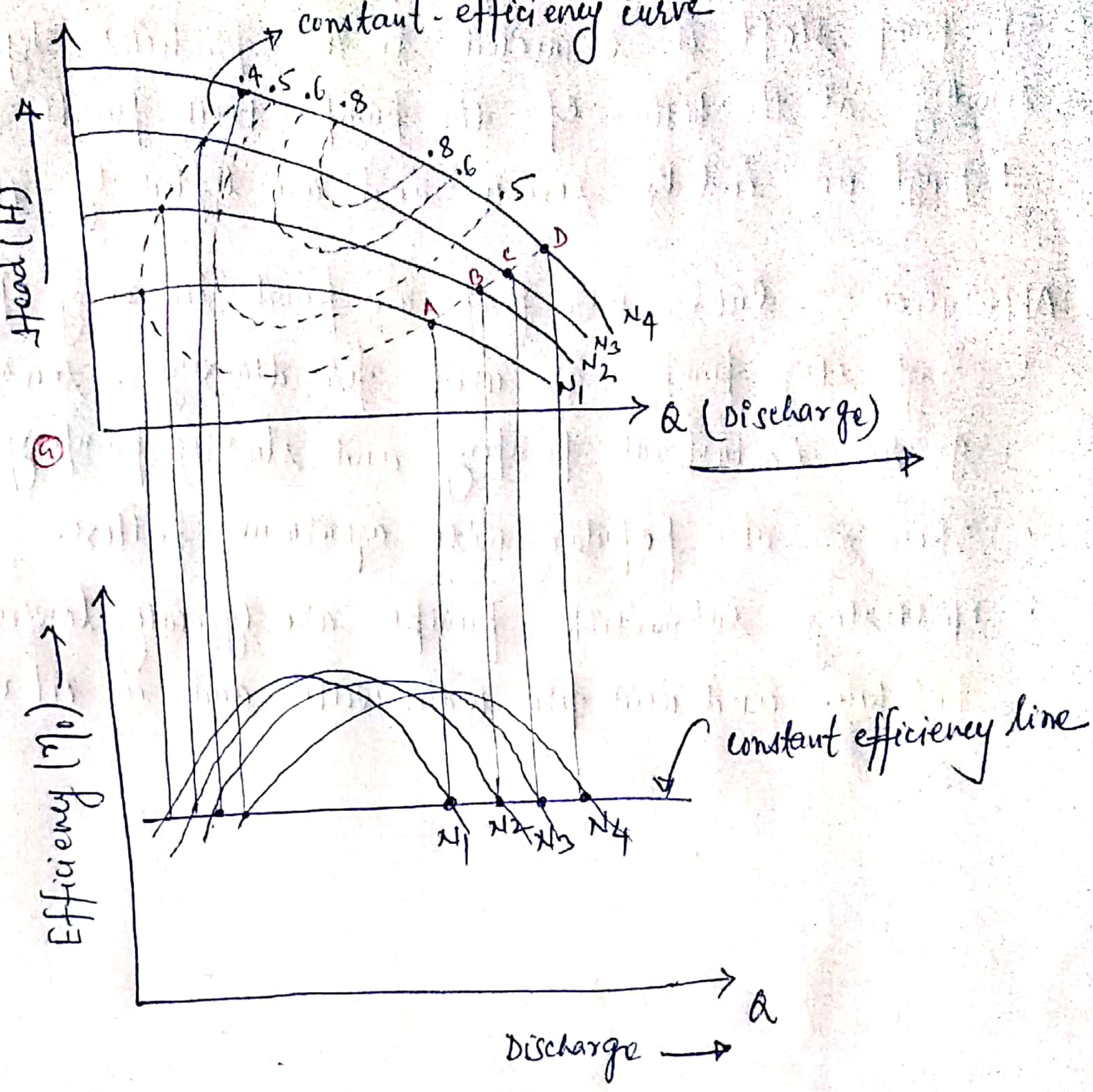


2. Operating characteristics curve —

At constant speed
N = constant



3. constant efficiency curve :- (or Iso-efficiency curve)



* Submersible pump :- It is a pump which has a sealed motor closed coupled to the pump body. The whole assembly is submerged in the fluid to be pumped.

- A submersible water pump pushes water to the surface, instead of sucking the water out of the ground.
- It is multistage centrifugal pumps operating in a vertical position.
- produced liquids, after being subjected to great centrifugal forces caused by the high rotational speed of the impeller, lose

~~this~~ their kinetic energy in the diffuser where a conversion of kinetic to pressure energy takes place.

- The pump shaft is connected to the protector by a mechanical coupling at the bottom of the pump. Well fluids enter the pump through an intake screen and are lifted by the pump stages.

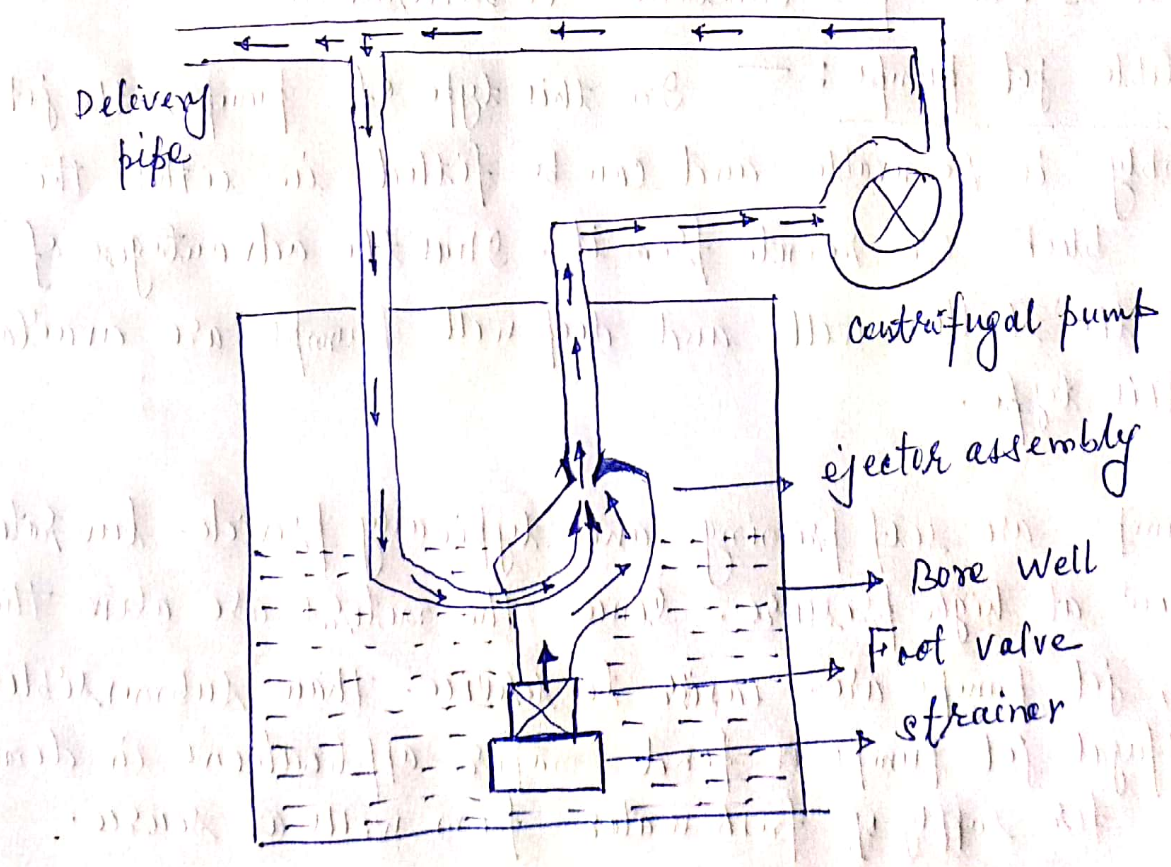
Application — Submersible pumps are found in many applications.

- Single stage pumps are used for drainage, sewage pumping, general industrial pumping and slurry pumping.
- These are also popular with aquarium filters.
- Multistage submersible pumps are typically lowered down a borehole and used for water wells and in oil wells.

Jet pumps

Jet pump is a generic name for pumps that use, to some extent, the suction created by a jet at a Venturi expansion to lift the pumping liquid.

Centrifugal jet pump :- This type of jet pump is a combination of a normal centrifugal pump and a jet device (called the injector) at the suction end. when the pump is started, a part of the water from delivery side of the pump is diverted in to a nozzle. water under high pressure is forced through this nozzle into the throat of a Venturimeter shaped pipe that is located in the suction side of the pump assembly. The negative pressure caused by the jet flow causes the water to be sucked up from the sump and delivers it to the pump. This additional suction enhances the total suction head of the pump assembly.



- Jet pumps are usually of three types —

1. <u>Shallow well jet pumps</u> : Here, the jet assembly is integral with the main centrifugal pump. Only one suction pipe leads from the sump/well into the pump. These pumps have a limitation in terms of suction head of about <u>5.5m</u> and hence the installation has to be within the elevation range of about 5m from the level of water in the sump.

2. <u>Deep well jet pumps</u> : In this the injector (i.e. the jet assembly) is remote from the pump and is installed in the well near the foot valve. In such type of pump two pipes lead from the pump to the well. One of them is feed from the pump to the jet and other is the delivery pipe. The injector assembly is attached to the suction pipe with a foot valve at the extreme end. The height of the jet assembly from the water surface of the well/sump is usually restricted to about 5.0m from cavitation considerations.

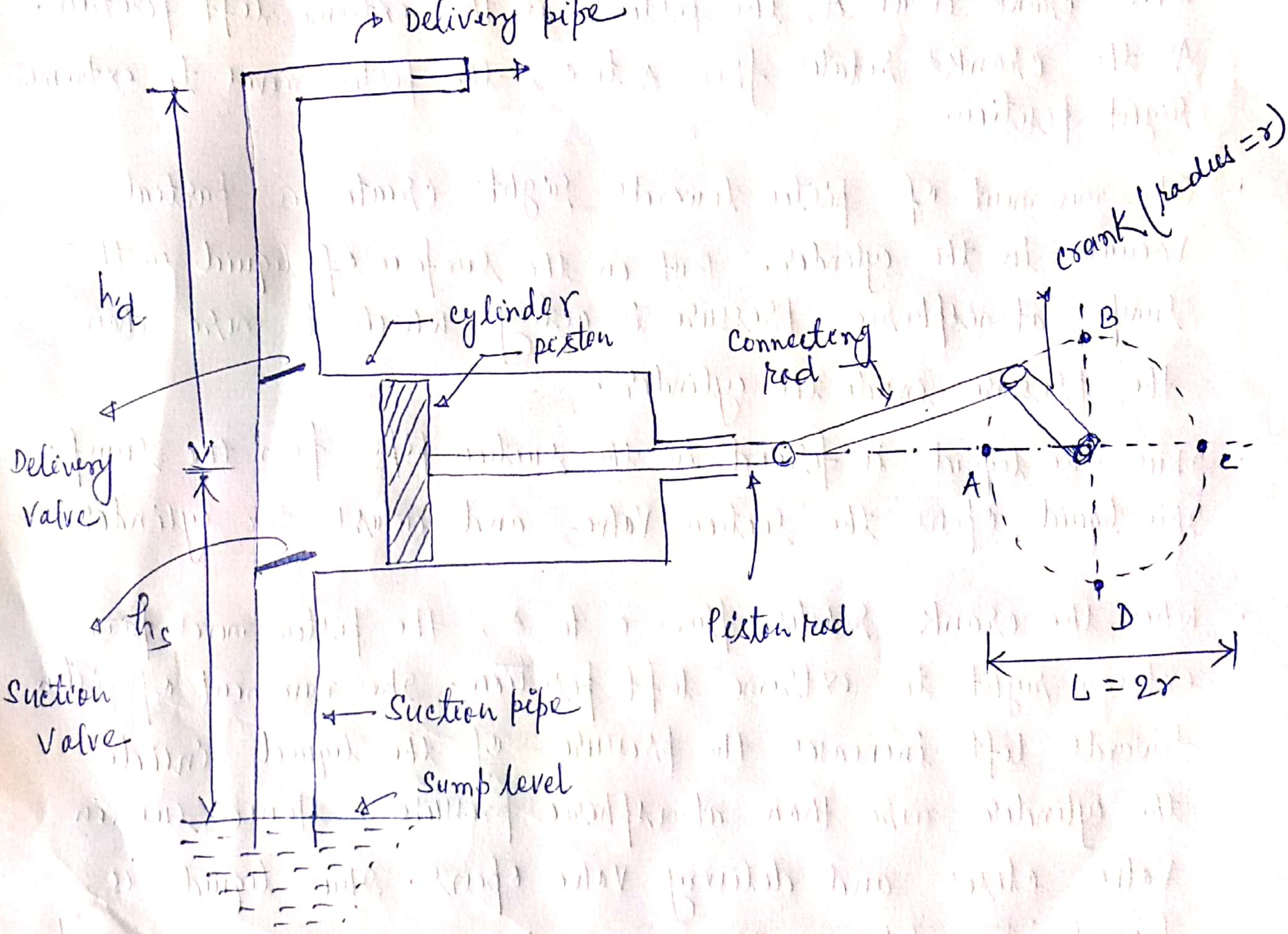
3. <u>Convertible jet pumps</u> :— In this type of pumps, the jet assembly is removable and can be fitted in either the pump block or remote from it. Thus the advantages of both the shallow well and deep well pumps are available in this type.

- Jet pumps are self priming and typically provide low rates of flow at high pressure. Since the motors are above the water, jet pumps are easier to service than submersibles. Centrifugal jet pumps find major applications in domestic usage for supply of fresh water from wells as source.

Reciprocating pump

- If the mechanical energy is converted into hydraulic energy (or pressure energy) by sucking the liquid into a cylinder in which a piston is reciprocating, which exerts the thrust on the liquid and increases its hydraulic energy, the pump is known as reciprocating pump.

* Construction of reciprocating pump :-



Main parts : ① A cylinder with a piston, piston rod, connecting rod and crank
② Suction pipe ③ Suction valve ④ Delivery pipe
⑤ Delivery valve.

Working:— Reciprocating pump consists of a piston which moves forwards and backwards in a closed cylinder. The movement of piston is obtained by by connecting the piston rod to a crank by means of a connecting rod. Crank is rotated by means of an electric motor.

- Suction and delivery pipes with suction valve and delivery valve are connected to the cylinder. These valves are non-return or one-way valves, which allow the water to flow in one direction only.

- When crank is at A, the piston is at the extreme left position. As the crank rotates from A to C, the piston moves to extreme right position.

- The movement of piston towards right creates a partial vacuum in the cylinder. But on the surface of liquid in the sump, atmospheric pressure is acting, which is more than the pressure inside the cylinder.

- Thus the liquid is forced in the suction pipe from the sump. This liquid opens the suction valve and enters the cylinder.

- When the crank rotates from C to A, the piston moves from extreme right to extreme left position. The movement of piston towards left increases the pressure of the liquid inside the cylinder more than atmospheric pressure. Hence suction valve closes and delivery valve opens. Thus liquid is forced into delivery pipe and is raised to a required height.

Analysis of reciprocating pump :

- The figure on page 1 is of single acting reciprocating pump.
- Single acting means the water is acting on one side of the piston only.

Let,
D = diameter of cylinder
A = Cross sectional area of cylinder or piston
$= \frac{\pi}{4} D^2$
r = radius of crank
N = R.P.M of crank
L = Length of stroke
h_s = suction head
h_d = delivery head.

Now, Volume of water delivered in one revolution or discharge of water in one revolution = (Area × length of stroke) = $A \times L$

Number of revolution per second $= \frac{N}{60}$

∴ Discharge of the pump per second, (Q)

$= $ (Discharge in one revolution) × (No. of revolution per second)

$$= AL \times \frac{N}{60} = \frac{ALN}{60}$$

- Weight of water delivered per second, $W = \rho g Q = \frac{\rho g ALN}{60}$

- Work done per second = (Weight of water lifted per second) × (Total height through which water is lifted)

$$= \boxed{\frac{\rho g ALN}{60} \times (h_s + h_d)}$$

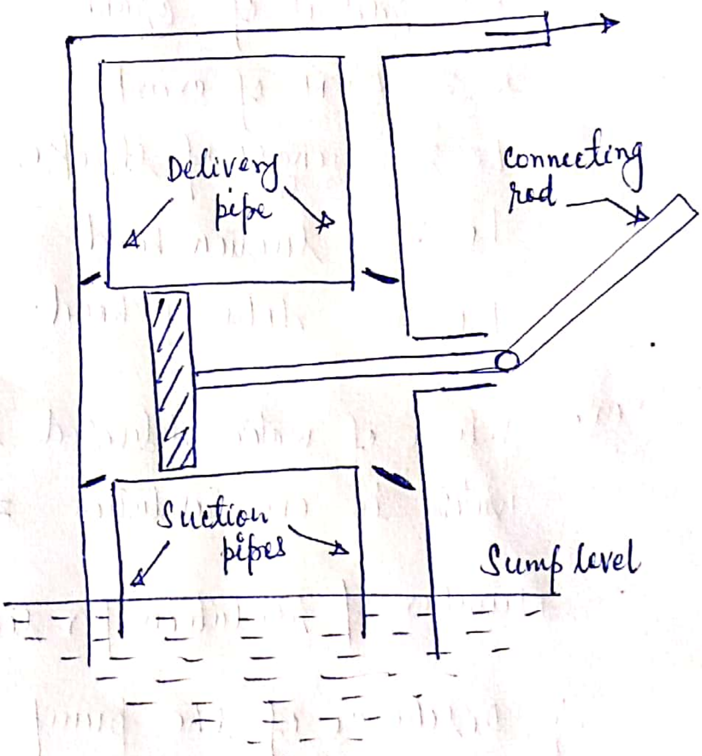
- Power required to drive the pump (in KW)

$$P = \frac{\text{Work done per second}}{1000} = \frac{\rho g \times ALN \times (h_s + h_d)}{60 \times 1000}$$

$$\boxed{P = \frac{\rho g \times ALN \times (h_s + h_d)}{60000}\ kW}$$

* Double-acting reciprocating pump :-

- In double acting reciprocating pump, water acts on both sides of the piston as shown. So, there are two suction pipes and two delivery pipes. When there is a suction stroke on one side of the piston, at the same time there is a delivery stroke on the other side of the piston.



- For one complete revolution of crank there are two delivery strokes.

Let D = diameter of the piston

d = diameter of piston rod

Area on one side of the piston, $(A) = \frac{\pi}{4} D^2$

Area on the other side of the piston, where the piston rod is connected – $A_1 = \frac{\pi}{4} D^2 - \frac{\pi}{4} d^2 = \frac{\pi}{4}(D^2 - d^2)$.

$\therefore$ Volume of water delivered in one revolution of crank

$$= (A \times \text{length of stroke}) + (A_1 \times \text{length of stroke})$$

$$= AL + A_1L = (A + A_1)L = \left[\frac{\pi}{4}D^2 + \frac{\pi}{4}(D^2 - d^2)\right]L$$

Discharge of pump per second (Q) $= \left(\begin{array}{c}\text{Volume of water delivered} \\ \text{in one revolution}\end{array}\right) \times \left(\begin{array}{c}\text{No. of revolution} \\ \text{per second}\end{array}\right)$

$$= \left[\frac{\pi}{4}D^2 + \frac{\pi}{4}(D^2 - d^2)\right] \times L \times \frac{N}{60}$$

if diameter (d) of connecting rod is very small, then it can be neglected,

So, $Q = \left(\frac{\pi}{4}D^2 + \frac{\pi}{4}D^2\right)\frac{LN}{60} = 2 \times \frac{\pi}{4}D^2 \times \frac{LN}{60} = \frac{2ALN}{60}$

This discharge is two times the discharge of single acting reciprocating pump.

- Work done per second $= (\text{Weight of water delivered}) \times \text{Total height}$

$$= \rho g \times Q \times h_{total}$$

$$= \rho g \times \frac{2ALN}{60} \times (h_s + h_d) = \boxed{2\rho g \times \frac{ALN}{60} \times (h_s + h_d)}$$

- Power required to drive the double-acting pump in kW,

$$P = \frac{\text{Work done per second}}{1000} = 2\rho g \times \frac{ALN}{60} \times \frac{h_s + h_d}{1000}$$

$$\boxed{P = \frac{2\rho g \times ALN \times (h_s + h_d)}{60{,}000}}$$

* Slip of reciprocating pump.

- It is defined as the difference between the theoretical discharge and actual discharge of the pump.
- The actual discharge of a pump is less than the theoretical discharge due to leakage.
- The difference of the theoretical discharge and actual discharge is known as slip of the pump.

$$\boxed{Slip = Q_{theor.} - Q_{actual}}$$

$$\text{Percentage slip} = \frac{Q_{th} - Q_{act.}}{Q_{th.}} \times 100 = \left(1 - \frac{Q_{act.}}{Q_{the}}\right) \times 100$$

$$= (1 - C_d) \times 100 \qquad \left[\because C_d = \frac{Q_{act}}{Q_{th.}}\right].$$

C_d → Co-efficient of discharge.

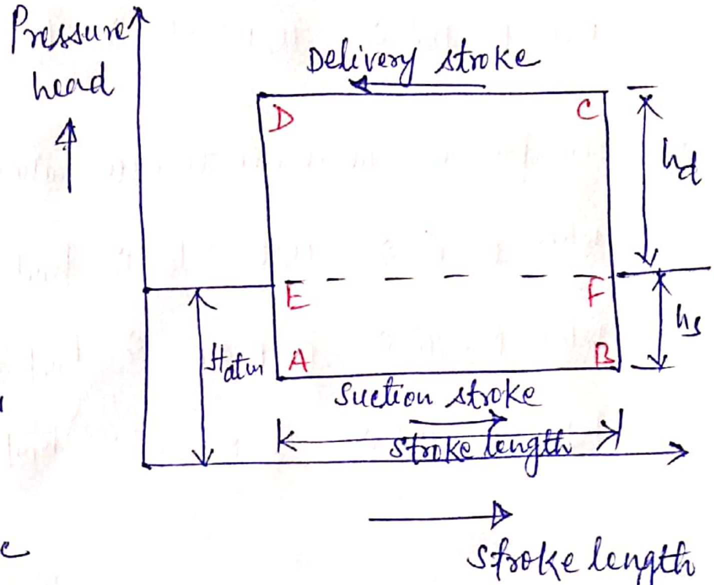
* Negative slip of reciprocating pump: –

- If the actual discharge is more than the theoretical discharge, the slip of the pump will become negative. In that case the slip of the pump is known as negative slip.
- Negative slip occurs when delivery pipe is short, suction pipe is long and pump is running at high speed.

Indicator diagram

- It is a graph between the pressure head in the cylinder and the distance travelled by piston from inner dead centre for one complete revolution of the crank.

Ideal indicator diagram :



- The graph between pressure head in the cylinder and stroke length of the piston for one complete revolution of the crank under ideal condition (means no acceleration and no friction head) is known as ideal indicator diagram.
- Line EF represents the atmospheric pressure head and equal to 10.3 m of water.
- During suction stroke, the pressure head in the cylinder is constant (h_s) and is below the atmospheric pressure head (H_{atm}) by a height of h_s. It is represented by line AB.
- During delivery stroke, the pressure head in the cylinder is constant (h_d) and is above the atmospheric pressure head (H_{atm}) by a height of h_d. It is represented by line CD.
- We know that, work done by pump per second $= \rho g \frac{ALN}{60} \times (h_s + h_d)$

Let, $\frac{\rho g AN}{60}$ = constant = K.

then work done per second $= K \times L(h_s + h_d)$

But $L(h_s + h_d)$ = area of indicator diagram.

So, work done per second = K (area of indicator diagram).

$\therefore$ $\boxed{\text{Work done per second} \propto \text{Area of indicator diagram.}}$

Effect of acceleration in suction and delivery pipes on indicator diagram :-

- The pressure head due to acceleration in the suction pipe $(h_{as}) = \frac{l_s}{g} \times \frac{A}{a_s} \omega^2 r \cos\theta$

when, $\theta = 0^\circ \Rightarrow \cos\theta = 1 \Rightarrow h_{as} = \frac{l_s}{g} \times \frac{A}{a_s} \omega^2 r$

when $\theta = 90^\circ \Rightarrow \cos\theta = 0 \Rightarrow h_{as} = 0$

when $\theta = 180^\circ \Rightarrow \cos\theta = -1 \Rightarrow h_{as} = -\frac{l_s}{g} \times \frac{A}{a_s} \omega^2 r.$

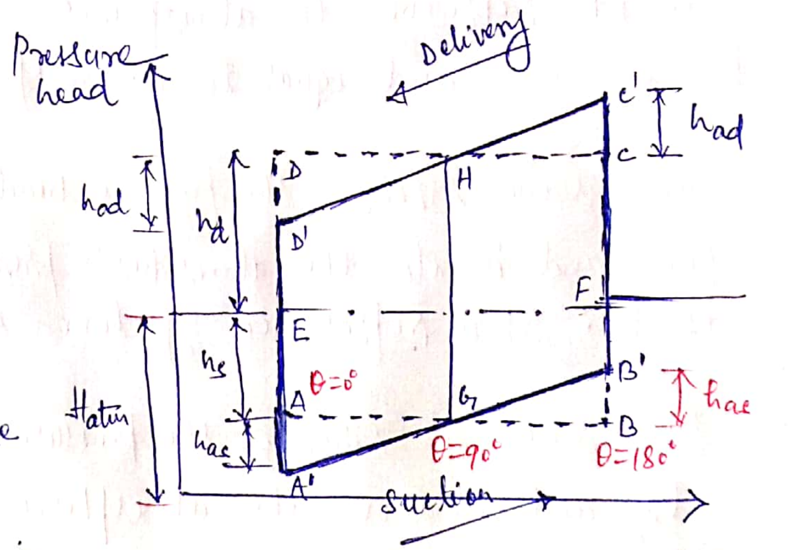
- The pressure head due to acceleration in the delivery pipe $(h_{ad}) = \frac{l_d}{g} \times \frac{A}{a_d} \omega^2 r \cos\theta$

when, $\theta = 0^\circ \Rightarrow \cos\theta = 1 \Rightarrow h_{ad} = \frac{l_d}{g} \times \frac{A}{a_d} \omega^2 r$

when, $\theta = 90^\circ \Rightarrow \cos\theta = 0 \Rightarrow h_{ad} = 0$

when, $\theta = 180^\circ \Rightarrow \cos\theta = -1 \Rightarrow h_{ad} = -\frac{l_d}{g} \times \frac{A}{a_d} \omega^2 r$

- Pressure head inside the cylinder during suction stroke will not be h_s rather it will the sum of h_s and h_{as}.



- At the begining of suction stroke $(\theta = 0^\circ)$, h_{as} = +ve, so, the pressure head in the cylinder will be $(h_s + h_{as})$ below the atmospheric pressure head.
- At the middle of the suction stroke, $(\theta = 0^\circ)$. h_{as} = zero. so, the pressure head in the cylinder is h_s below the atmospheric pressure head.
- At the end of suction stroke $(\theta = 180^\circ)$. so, (h_{as} = -ve), and hence, the pressure head in the cylinder will be $(h_s - h_{as})$ below the atmospheric pressure head.

- Similarly for delivery stroke, the pressure head inside the cylinder will not be h_d, rather it will be the sum of h_d and h_{ad}.
- At the beginning of the delivery stroke, h_{ad} is positive and hence the pressure head in the cylinder will be $(h_d + h_{ad})$ above the atmospheric pressure head.
- At the middle of the delivery stroke, $h_{ad} = 0$, hence, pressure head in the cylinder will be h_d above the atmospheric pressure head.
- At the end of delivery stroke, $h_{ad} = -ve$, and hence pressure in the cylinder will $(h_d - h_{ad})$ above the atmospheric pressure head.
- Due to acceleration effect in suction and delivery pipe, the indicator diagram has changed from ABCD to A'B'C'D'. But the area of indicator diagram A'B'C'D' = Area of indicator diagram ABCD. Hence, work done by the pump on water remains same.

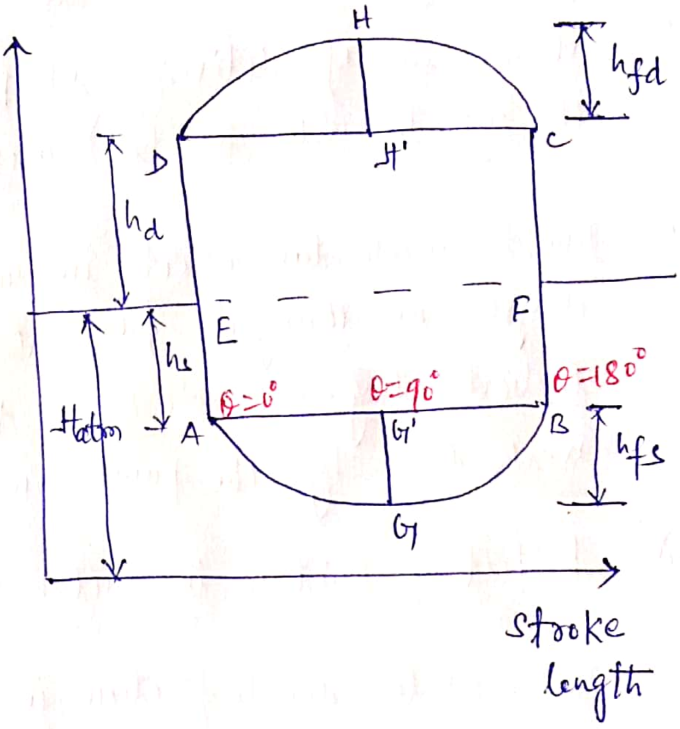
* Effect of friction in suction and delivery pipes on indicator diagram:-

- The head loss due to friction in suction pipe is $h_{fs} = \frac{4fl_s}{d_s \times 2g}\left(\frac{A}{a_s}\omega r \sin\theta\right)^2$
- The head loss due to friction in delivery pipe $h_{fd} = \frac{4fl_d}{d_d \times 2g}\left(\frac{A}{a_d}\omega r \sin\theta\right)^2$
- Now, pressure head inside cylinder during suction stroke will not be equal to h_s, rather it will be the sum of $(h_s + h_{fs})$ due to presence of friction.
- Similarly, pressure head inside cylinder during delivery stroke will not be equal to h_d, rather it will be the sum of $(h_d + h_{fd})$ due to presence of friction.
- At the beginning of the suction or delivery stroke, $\theta = 0°$, hence, $\sin\theta = 0$. This means h_{fs} and $h_{fd} = 0$.
- At the middle of the suction or delivery stroke, $\theta = 90°$, hence, $\sin\theta = 1$.

$\Rightarrow h_{fs} = \frac{4 f l_s}{d_s \times 2g}\left(\frac{A}{a_s}\omega r\right)^2$, $h_{fd} = \frac{4 f l_d}{d_d \times 2g} \times \left(\frac{A}{a_d}\omega r\right)^2$

- At the end of suction stroke or delivery stroke, $\theta = 180^\circ \Rightarrow \sin\theta = 0$
$\Rightarrow h_{fs} = 0$ and $h_{fd} = 0$.

- Variation of h_{fs} and h_{fd} with θ is parabolic in nature. So, indicator diagram during suction and delivery stroke with friction in suction and delivery pipe will be shown.



- Area of parabolas AGB and CHD represents the work done against friction in suction and delivery pipes.

- Area AGB $= AB \times \frac{2}{3} GG'$

$= AB \times \frac{2}{3} h_{fs}$

$= L \times \frac{2}{3} h_{fs}$,

Similarly Area CHD $= L \times \frac{2}{3} h_{fd}$

* Effect of acceleration and friction in suction and delivery pipes on indicator diagram :-

Figure in next page shows the combined effect of acceleration and friction in suction in delivery pipes.

- At the beginning of the suction stroke, $\theta = 0^\circ$ hence, $h_{as} = \frac{l_s}{g} \times \frac{A}{a_s} \omega^2 r$ and $h_{fs} = 0$. Thus the pressure head in the cylinder will be $(h_s + h_{as})$ below the atmospheric pressure head.

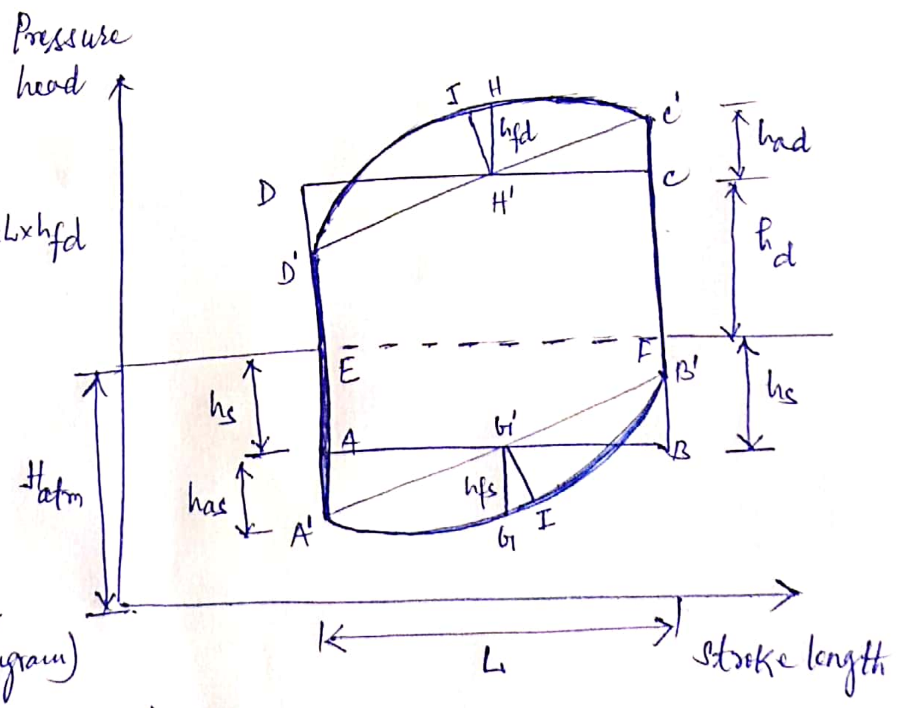
- At the middle of the suction stroke $h_{as}=0$. but $h_{fs}=\frac{4fl_s}{d_s \times 2g} \times \left(\frac{A}{a_s} \omega r\right)^2$.
Thus, the pressure head in the cylinder will be (h_s+h_{fs}) below the atmospheric pressure head.

- At the end of suction stroke, $h_{as}=\frac{-l_s}{g} \times \frac{A}{a_s} \omega^2 r$, but $h_{fs}=0$.
Thus, the pressure head in the cylinder will be (h_s-h_{as}) below the atmospheric head.

- At the begining of the delivery stroke, $h_{ad}=\frac{l_d}{g} \times \frac{A}{a_d} \times \omega^2 r$, but $h_{fd}=0$,
Thus the pressure head in the cylinder will be (h_d+h_{ad}) above the atmospheric pressure head.

- In the middle of the delivery stroke, $h_{ad}=0$, and $h_{fd}=\frac{4fl_d}{d_d \times 2g}\left(\frac{A}{a_d} \omega r\right)^2$.
Thus the pressure head in the cylinder will be (h_d+h_{fd}) above from the atmospheric pressure head.

- At the end of the delivery stroke, $h_{ad}=\frac{-l_d}{g} \times \frac{A}{a_d} \times \omega^2 r$, and $h_{fd}=0$.
Thus the pressure head in the cylinder will be (h_d-h_{ad}) above the atmospheric pressure head.



Here, area of indicator diagram
$= (h_s+h_d) \times L + \frac{2}{3} \times L \times h_{fs} + \frac{2}{3} \times L \times h_{fd}$
$= \left(h_s+h_d+\frac{2}{3}h_{fs}+\frac{2}{3}h_{fd}\right) \times L$

Now, (Work done by pump)
$\propto$ (Area of indicator diagram)
$= KL\left(h_s+h_d+\frac{2}{3}h_{fs}+\frac{2}{3}h_{fd}\right)$,

$K=\frac{\rho g A N}{60}$ ... for a single acting pump
$=\frac{2\rho g AN}{60}$ ... for a double " "

20.9 AIR VESSELS

An air vessel is a closed chamber containing compressed air in the top portion and liquid (or water) at the bottom of the chamber. At the base of the chamber there is an opening through which the liquid (or water) may flow into the vessel or out from the vessel. When the liquid enters the air vessel, the air gets compressed further and when the liquid flows out the vessel, the air will expand in the chamber.

An air vessel is fitted to the suction pipe and to the delivery pipe at a point close to the cylinder of a single-acting reciprocating pump :

(i) to obtain a continuous supply of liquid at a uniform rate,

(ii) to save a considerable amount of work in overcoming the frictional resistance in the suction and delivery pipes, and

(iii) to run the pump at a high speed without separation.

Fig. 20.9 shows the single-acting reciprocating pump to which air vessels are fitted to the suction and delivery pipes. The air vessels act like an intermediate reservoir. During the first half of the suction stroke, the piston moves with acceleration, which means the velocity of water in the suction pipe is more than the mean velocity and hence the discharge of water entering the cylinder will be more than the mean discharge. This excess quantity of water will be supplied from the air vessel to the cylinder in such a way that the velocity in the suction pipe below the air vessel is equal to mean velocity of flow. During the second half of the suction stroke, the piston moves with retardation and hence velocity of flow in the suction pipe is less than the mean velocity of flow. Thus, the discharge entering the cylinder will be less than the mean discharge. The velocity of water in the suction pipe due to air vessel is equal to mean velocity of flow and discharge required in cylinder is less than the mean discharge. Thus, the excess water flowing in suction pipe will be stored into air vessel, which will be supplied during the first half of the next suction stroke.

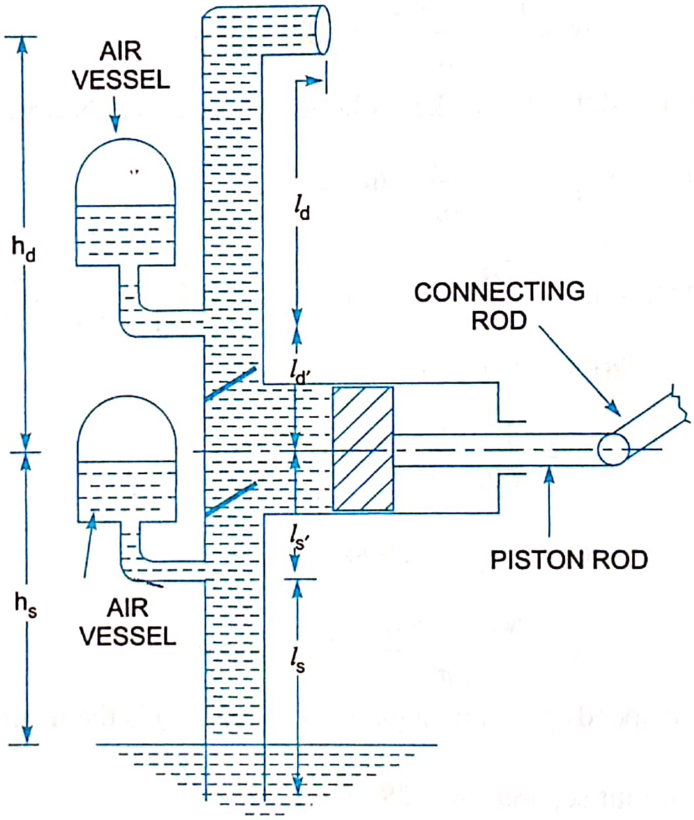


Fig. 20.9 *Air vessels fitted to reciprocating pump.*

When the air vessel is fitted to the delivery pipe, during the first half of delivery stroke, the piston moves with acceleration and forces the water into the delivery pipe with a velocity more than the mean velocity. The quantity of water in excess of the mean discharge will flow into the air vessel. This will compress the air inside the vessel. During the second half of the delivery stroke, the piston moves with retardation and the velocity of water in the delivery pipe will be less than the mean velocity. The water already stored into the air vessel will start flowing into the delivery pipe and the velocity of flow in the delivery pipe beyond the point to which air vessel is fitted will become equal to the mean velocity. Hence, the rate of flow of water in the delivery pipe will be uniform.